Performance of Steady-State Voltage Stability Analysis in MATLAB Environment

Jan Veleba [1]

[1] University of West Bohemia in Pilsen, Regional Innovation Centre of Electrical Engineering, Pilsen, Czech Republic, e-mail: *jveleba@rice.zcu.cz*

Abstract — **In recent years, electric power systems have been often operated close to their working limits due to increased power consumptions and installation of renewable power sources. This situation poses a serious threat to stable network operation and control. Therefore, voltage stability is currently one of key topics worldwide for preventing related black-out scenarios. In this paper, modelling and simulations of steady-state stability problems in MATLAB environment are performed using author-developed computational tool implementing both conventional and more advanced numerical approaches. Their performance is compared with the Simulink-based library Power System Analysis Toolbox (PSAT) in terms of solution accuracy, CPU time and possible limitations.**

Keywords — Steady-state voltage stability, continuation load flow analysis, predictor-corrector method, voltage stability margin, voltage-power sensitivity, Power System Analysis Toolbox

I. Introduction

Steady-state voltage stability is defined as the capability of the system to withstand a small disturbance (e.g. fault occurrence, small change in parameters, topology modification, etc.) without abandoning a stable operating point [1]-[5]. Voltage stability problems are generally bound with long "electrical" distances between reactive power sources and loads, low source voltages, severe changes in the system topology and low level of var compensation. However, this does not strictly mean that voltage instability is directly connected only with low voltage scenarios. Voltage collapse can arise even during normal operating conditions (e.g. for voltages above nominal values). Moreover, variety of practical situations can eventually lead to voltage collapse, e.g. tripping of a parallel connected line during the fault, reaching the var limit of a generator or a synchronous condenser, restoring low supply voltage in induction motors after the fault. All these cause the reduction of delivered reactive power for supporting bus voltages followed by increases of branch currents and further voltage drops to even lower reactive power flow or line tripping until the voltage collapse occurs. This entire process may occur in a rather large time frame from seconds to tens of minutes.

To prevent voltage collapse scenarios, several types of compensation devices are massively used worldwide - both shunt capacitors/inductors, series capacitors, SVCs, synchronous condensers, STATCOMs, etc. To reduce voltage profiles (in case of low demand), var consumptions must be increased by switching in shunt reactors, disconnecting cable lines (if possible), reducing voltage-independent MVAr output from generators and synchronous condensers, etc. To increase bus voltages, opposite corrective actions are to be taken. These include reconfigurations (connecting parallel lines / transformers / cables), power transfer limitations and activations of new generating units at most critical network areas.

Furthermore, the voltage load shedding of low-priority loads (usually by 5, 10 or 20 % in total) is usually realized at subtransmission substations using undervoltage relays. These relays work similarly as on-load tap-changing (OLTC) transformers. They are activated by long-term voltage dips (in region between 0.8 and 0.9 pu) and as the result, they trip the load feeders - typically in steps of 1 to 2 % of the load at any given time (with time delays of 1-2 minutes after the voltage dip). The larger voltage dip, the faster and larger response of the relay [2].

Low voltage profiles are usually averted by actions of OLTC transformers. However, each tap position corresponds to an increase of the load which eventually leads to higher branch losses and further voltage drops [1]-[2],[4]. Therefore, OLTC transformers should be blocked during low voltage stability scenarios. Negative effects of OLTC actions during low voltage conditions are presented in many studies with voltage stability margin calculation from synchrophasor measurements [6]-[7].

This paper is organized as follows. Chapters II and III describe conventional Cycled Newton-Raphson (N-R) and more robust Continuation Load Flow (CLF) methods for the voltage stability analysis, respectively. Independent tool - Power System Analysis Toolbox (PSAT) - is briefly introduced in Chapter IV. In Chapter V, key properties of both of the author-developed codes are discussed. Chapters VI and VII show the results of individual approaches when solving voltage stability of a broad variety of test power systems. Chapter VIII closes the paper with some concluding remarks and the evaluation of each technique applied.

II. Conventional Numerical Calculation Of The Voltage Stability Problem

When increasing the loading (or loadability factor λ) of the system, its bus voltages slowly decrease due to the lack of reactive power. At the critical point (called singular or bifurcation), characterized by maximum loadability factor λ_{max} and critical bus voltages, the system starts to be unstable and voltage collapse appears (system black-out). From this point on, only lower loading with low voltage values leads to the solution. The dependence between bus voltage magnitudes and λ is graphically represented by the V-P curve (also referred to as the nose curve). Unfortunately, the current (so-called base-case) position of the system operating point on the V-P curve is not known along with its distance from the voltage collapse (so-called voltage stability margin). Thus, location of the singular point must be found during the voltage stability analysis.

Note: Values of λ_{max} and critical voltages are rather theoretical since they do not reflect voltage/flow limits of network buses/branches. When incorporating these practical restrictions, the real maximum loadability λ_{max}^* can be found (i.e. the maximum value for keeping all network buses and branch loadings within limits).

Traditional approach for finding the maximum system loadability is to apply the standard N-R method [8] for the base-case load flow calculation (i.e. for λ = 1.0). When obtaining current position on the V-P curve, network loading (i.e. loads/generations in selected network buses) is increased in defined manner by a certain step and the load flow is computed repetitively along with a new position on the V-P curve. This process continues in an infinite loop until the singular point is reached. However, total number of iterations in each V-P step is gradually increasing so that when close to the singular point, the N-R method fails to converge, i.e. no solution is provided. This relates to the fact, that Jacobian $\underline{J}$ becomes singular (i.e. det $\underline{J} \approx 0$) and its inverse matrix cannot be computed for successful numerical convergence.

For speeding up the calculation, a variable step change is applied. Usually, a single default step value is used. When obtaining the divergence of the N-R method, the step size is simply divided by two and the calculation for the current V-P point is repeated until the convergence is achieved. When the current step size value reaches the pre-set minimum value, the calculation is stopped. Despite of the relatively simple procedure, the Cycled N-R method enables the completion of the stable V-P part only. The unstable part including the singular point cannot be examined. Also, high CPU requirements prevent this method from being employed for larger power systems.

In this paper, the Cycled N-R algorithm was developed and further tested on wide range of test power systems.

III. CONTINUATION LOAD FLOW ANALYSIS

The CLF analysis [1],[9] suitably modifies conventional load flow equations to become stable also in the bifurcation point and therefore being capable of drawing both upper/lower parts of the V-P curve. It uses a two-step predictor/corrector algorithm along with the new unknown state variable called continuation parameter (CP).

The predictor (1) is a tangent extrapolation of the current operation point estimating approximate position of the new point on the V-P curve.

$$\begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}^{\text{predicted}} = \begin{bmatrix} \theta_0 \\ V_0 \\ \lambda_0 \end{bmatrix} + \sigma \begin{bmatrix} & & \vdots & \\ & \underline{J} & \vdots & K \\ & & \vdots & \\ \cdots & \cdots & \cdots & \cdots \\ & & e_k & \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (1)$$

Vector K contains base-case power generations and loads. Variables θ_0, V_0, λ_0 define the system state from the previous corrector step. The vector e_k is filled with zeros and certain modifications (see [1],[9]) are implemented for the selected CP in each network bus k at the current point on the V-P curve. Remaining elements in (1) are the newly computed Jacobian $\underline{J}$ and step size σ of the CP.

The tangent predictor is relatively slow, anyway shows good behaviour especially in steep parts of the V-P curve. Unlike the tangent predictor, secant predictor is simpler, computationally faster and behaves well in flat parts of the V-P curve. In steep parts (i.e. close to the singular point and at sharp corners when a generator exceeds its var limit) it computes new predictions too far from the exact solution. This may eventually lead to serious convergence problems in the next corrector step. Therefore, the tangent predictor is more recommended to be applied.

The corrector is a standard N-R algorithm for correcting state variables from the predictor step to satisfy load flow equations. Due to one extra parameter λ, additional condition (2) must be included for keeping the value of the CP constant in the current corrector step. This condition makes the final set of equations non-singular even at the bifurcation point.

$$x_k - x_k^{\text{predicted}} = 0 \quad , \; x = \begin{cases} \lambda & \text{if CP is } \lambda \\ V & \text{if CP is } V \end{cases} \quad (2)$$

Difference between both types of predictors and the entire process of the predictor/corrector algorithm is graphically demonstrated in Fig. 1. Horizontal/vertical corrections are performed with respect to the chosen CP type.

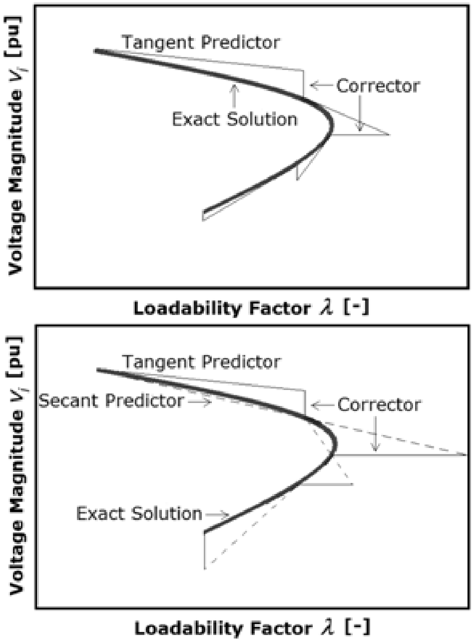

Fig. 1. Predictor/Corrector Mechanism for the CLF Analysis [10].

As the CP, state variable with the highest rate of change must be chosen (i.e. λ and V in flat and steep parts of the V-P curve, respectively). When the process starts diverging, parameter σ must be halved or parameter CP must be switched from λ to V.

The step size should be carefully increased to speed-up the calculation when far from the singular point or decreased to avoid convergence problems when close to the peak. The step size modification based on the current position on the V-P curve (i.e. as a function of the line slope for previous two corrected points on the V-P curve) is recommended in [11]. This approach belongs to so-called rule-based or adaptive step size control algorithms.

In [10], several voltage stability margin indices ($VSMI_i$, $VSMI_{ik}$) are presented along with relative var reserve coefficient and voltage-load sensitivity factors (VSF_i) for comprehensive voltage stability analysis and location of weak or sensitive system buses/branches/areas. In these regions, preventive or remedial actions should be taken. Procedures for allocating individual compensation devices and possible effects are also discussed.

The CLF analysis still remains very popular for high-speed solving of voltage stability studies. Due to its reliable numerical behaviour, it is often included into the N-R method providing stable solutions even for ill-conditioned load flow cases. Moreover, it is applied in foreign control centres for N-1 on-/off-line contingency studies with frequencies of 5 and 60 minutes [2], respectively.

IV. Power System Analysis Toolbox (PSAT)

PSAT [12] is a Simulink-based open-source library for electric power system analyses and simulations. It is distributed via the General Public License (GPL), its download and use is free of charge. However, there is no warranty that the Toolbox will provide correct and accurate results. All corrections and possible repairs or improvements are to be done on the customer side.

It contains the tools for Power Flow (busbars, lines, two-/three-winding transformers, slack bus(es), shunt admittances, etc.), CLF and OPF data (power supply/demand bids and limits, generator power reserves and ramping data), Small Signal Stability Analysis and Time Domain Simulations. Moreover, line faults and breakers, various load types, machines, controls, OLTC transformers, FACTS and other can be also modelled. User defined device models can be added as well.

All studies must be formulated for one-line network diagram only - either in input data *.m file in required format or in graphical *.mdl file, where the schema is manually drawn. For the former option, input data conversions from and to various common formats (PSS/E, DIgSILENT, IEEE cdf, NEPLAN, PowerWorld and more others) are available.

When compared to another MATLAB-based open-source tool MATPOWER [13], PSAT is more efficient and highly advanced by providing more analyses, problem variations, possible outputs and other useful features in its user-friendly graphical interface. MATPOWER does not support most of advanced network devices, entirely omits CLF analysis and has no graphical user interface or graphical network construction ability. Also, it does not consider var limits in PV buses. Incorrect interpretation of reactive power branch losses can be also observed.

V. Properties of Author-Developed Codes in MATLAB Environment

Both Cycled N-R and CLF procedures were developed in MATLAB environment for providing fundamental examination of medium-sized and larger power systems in terms of steady-state voltage stability. Several key aspects of these codes are discussed below.

1] Predictor: Despite of computationally more complex algorithm, the tangent predictor was used for finding reliable estimations of new V-P points especially around the singular point. It is applied in CLF algorithm only.

2] Corrector: First, a corrector step is used at the start of the CLF program to find the base-case point for further calculations. Due to possible weak numerical stability at this point (for badly-scaled power systems), the One-Shot Fast-Decoupled (OSFD) procedure is implemented to the standard N-R method for providing more stable solutions and thus preventing numerical divergence. Moreover, voltage truncation (SUT algorithm) is also included into the state update process at every N-R's iteration. Both of these stability approaches were introduced in [14] and further tuned and tested in [15]. Both were also applied to the Cycled N-R algorithm to increase the loading range, for which the stable load flow solutions can be obtained (i.e. closer proximity to the singular point can be reached).

3] Step size: Largest-load PQ network bus is chosen for computing the angle α between the horizontal and the line interconnecting two adjacent V-P points. Based on this, the step size evaluation function (3) is applied - see Fig. 2.

$$\sigma = \begin{cases} \sigma_\mathrm{L} & \text{for} \quad |\alpha| \geq \pi/8 \\ \sigma_\mathrm{U} & \text{for} \quad |\alpha| \leq \pi/32 \\ A/\sin^2\alpha + B & \text{otherwise} \end{cases} \tag{3}$$

The upper and lower step limit constants σ_U and σ_L define the step size for the flat part of the V-P curve and for close vicinity to the singular point, respectively.

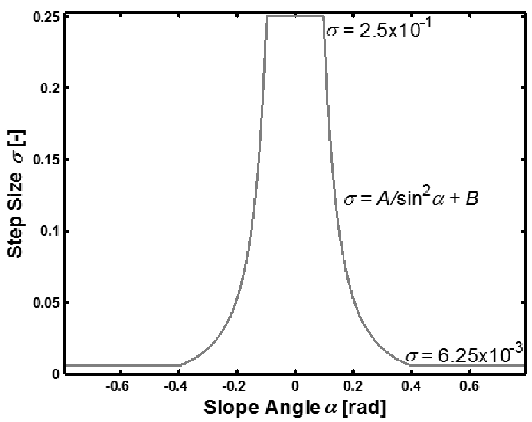

Fig. 2. Step size evaluation function [10].

For the Cycled N-R algorithm, this is a rather too complex concept of the step size control. Therefore, only a single step size is chosen at the start and a simple step-cutting technique (dividing by 2) is applied in case of divergence.

4] Ending criterion: Only stable part of the V-P curve (incl. exact singular point calculation) is computed by the CLF code. Thus, if the computed value of λ begins to decrease, the process is stopped. For the Cycled N-R code, the calculation is terminated when the step size falls below a certain small value (e.g. 1×10^{-8}). For each load flow case, maximum number of iterations and permitted tolerance for convergence is set to 20 and 1×10^{-8}, respectively.

5] Calculation speed and accuracy: For excessively accurate voltage stability solutions, values of σ_U and σ_L 2.5×10^{-2} and 6.25×10^{-4} are used in the CLF algorithm. Rather compromise values of 5×10^{-2} and 1×10^{-2} are also used to obtain fast and fairly accurate solutions for any of tested power systems. For the Cycled N-R algorithm, initial step size of 2.5×10^{-2} is chosen as sufficient.

6] Code versatility: Both Cycled N-R and CLF procedures are programmed so that the user directly specifies an arbitrary group of network buses for an load/generation increase. From this set of buses, only those non-slack buses with non-zero active power loads/generations are activated for the analysis. In all studies performed, a load/generation increase in the entire network (i.e. all network buses selected) is considered.

Two scenarios can be activated by the user. a) L scenario increases both P/Q loads in selected PQ/PV buses with a constant power factor (i.e. with identical increase rate). b) L+G scenario increases both P/Q loads in selected PQ/PV buses and P generations in selected PV buses (with identical increase rate).

7] Var limits: In both approaches, bus-type switching logics are applied to iteratively computed reactive powers Q_{Gi} in PV buses when exceeding the var limit (4) or to relevant bus voltages when returning the vars back inside the permitted var region (5). Symbol $^{(p)}$ denotes the current iteration number.

$$Q_{Gi}^{(p)} = \begin{cases} Q_{Gi\max} & \text{if} \quad Q_{Gi}^{(p)} > Q_{Gi\max} \\ Q_{Gi\min} & \text{if} \quad Q_{Gi}^{(p)} < Q_{Gi\min} \end{cases} \tag{4}$$

$$V_i = V_i^{sp} \ \text{if} \left\{ \begin{matrix} Q_{Gi}^{(p)} = Q_{Gi\max} \ \text{AND} \ V_i > V_i^{sp} \\ \text{OR} \\ Q_{Gi}^{(p)} = Q_{Gi\min} \ \text{AND} \ V_i < V_i^{sp} \end{matrix} \right\} \tag{5}$$

The terms Q_{Gimax} and Q_{Gimin} are the upper and lower var limits, the term V_i^{sp} determines the specified value of the voltage magnitude for each PV bus.

8] Code limitations: a) With increased loading, lower/upper var limits in PV buses should not be fixed but variable proportionally to the generated active power. In both codes, constant var limits are used for more pessimistic V/Q control. b) Only identical increase rate is applied. However, implementing user-defined increase rates for each load/generation would not pose any serious problem.

9] Sparse programming: Sparsity techniques along with smart vector/matrix programming are used in both Cycled N-R and CLF codes to significantly decrease the CPU time needed for each load flow case.

10] Outputs: Theoretical value of λ_{max} and V-λ data outputs for V-P curves are computed and stored or graphically projected. Respective values of λ for switching some of PV buses permanently to PQ are also recorded. Voltage and power flow limits were not considered for the evaluation of the real maximum loadability λ_{max}^*.

VI. Testing of Cycled N-R and CLF Algorithms for Solving Voltage Stability Load Flow Problems

Total number of 50 test power systems between 3 and 734 buses were analyzed using developed Cycled N-R and CLF algorithms in the MATLAB environment. Identical increase rate was applied to all network buses (before filtering those with non-zero active power loads or generations). For both L and L+G scenarios, only stable part of the V-P curve was calculated with included var limits. Settings of both codes are as introduced in Chapter V, paragraphs 4] and 5]. In Tab. I., voltage stability solutions of several test cases are shown. Presented results contain the maximum loadability, numbers of stable V-P points and CPU times in seconds needed.

For each case, the first two rows show the outputs of the CLF code for excessive accuracy and compromise accuracy, respectively. For comparison purposes, the third row provides the results of the Cycled N-R code.

TABLE I.
VOLTAGE STABILITY SOLUTIONS USING CYCLED N-R AND CLF ALGORITHMS - L AND L+G SCENARIOS

Case	Scenario L			Scenario L+G		
	λ_{max} [-]	pts	CPU [s]	λ_{max} [-]	pts	CPU [s]
IEEE9II	1.302632	331	0.5616	1.162053	215	0.3900
	1.302632	27	0.1404	1.162052	24	0.1248
	1.302632	23	0.4056	1.162053	20	0.4212
IEEE14	1.760331	658	1.2012	1.777995	506	0.9360
	1.760331	87	0.2340	1.777995	59	0.2028
	1.760331	43	0.5460	1.777995	45	0.6396
IEEE30	1.536905	854	1.9500	1.546751	726	1.6536
	1.536905	88	0.2808	1.546752	124	0.4212
	1.536905	37	0.6396	1.546751	37	0.6552
IEEE57	1.406778	891	2.9016	1.616845	399	1.3884
	1.406778	229	0.6864	1.616845	57	0.2652
	1.406778	27	0.8112	1.616845	37	0.8112
IEEE162	1.079959	1640	12.9169	1.138996	1185	9.3913
	1.079960	464	3.1044	1.138996	65	0.8112
	1.079960	13	1.7628	1.138996	16	1.8408
IEEE300	1.024573	8457	103.8655	1.058820	311	4.0092
	1.024573	529	7.0044	1.058819	94	1.4508
	1.024573	16	2.4180	1.058820	17	2.5584
EPS734II	3.104162	139	4.5864	3.104162	139	4.8360
	3.104083	46	1.8720	3.104083	46	1.8408
	3.104162	96	8.2369	3.104162	96	8.1745

As can be seen, exact solutions of maximum loadability were obtained for both of tested methods and each of the three accuracy settings. The first setting was definitely too much focused on producing exact results. Therefore, numbers of V-P points and CPU times were pushed often above 200 and 1 second, respectively. When using fair compromise setting, the maximum error for λ_{max} from all 50 test power systems was only 0.0185 percent, while numbers of points and CPU times were decreased on average by 75.27 percent and 64.11 percent, respectively.

The Cycled N-R code obtains highly accurate results in terms of solution accuracy. In majority of cases, it provides even better solutions than CLF algorithm with compromise accuracy. Surprisingly, it always computes slightly higher maximum loadability values than by the high-accurate CLF code. This seems to be one visible drawback of the Cycled N-R method. Only low numbers of V-P points are needed for reaching close proximity to the singular point. These numbers are well comparable to those needed for the compromise CLF code.

Unfortunately, each divergence case (between 22 and 28) significantly delays the entire computation process of the Cycled N-R method. Therefore, the Cycled N-R code suffers from being extremely time-dependent on computing of each V-P point. When compared with the compromise CLF code, the CPU time needed by the Cycled N-R method is on average about 167 % higher.

Therefore, the compromise CLF code seems to be the best method for providing fast and highly accurate voltage stability solutions.

Stable V-P curves of the IEEE 30-bus power system (L+G scenario) are computed by both Cycled N-R and CLF methods and shown in Figs. 3 and 4, respectively. For the CLF method, the V-P curves are extended to demonstrate numerical stability of the CLF algorithm around the singular point. Extension of V-P curves in the unstable region is provided for $0.97\times\lambda_{max} < \lambda < \lambda_{max}$.

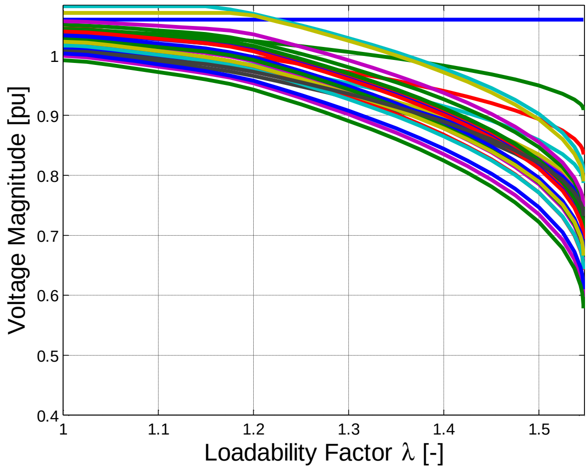

Fig. 3. V-P curves for the IEEE 30-bus system (Cycled N-R method).

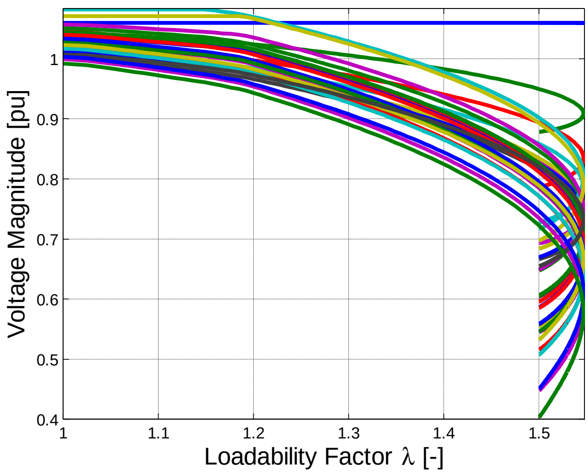

Fig. 4. Extended V-P curves for the IEEE 30-bus system (CLF method).

As Tab. I. indicates, applied version of the CLF method is still not applicable for real-time voltage stability monitoring, but it can be useful for off-line reliability, evaluation or planning studies of even larger networks.

VII. Testing of PSAT for Solving Voltage Stability Load Flow Problems

Before solving voltage stability in PSAT, the load flow analysis of a system must be performed. Therefore, large number of load flow studies is solved using PSAT to detect any of its possible weaknesses. Results were compared with the author-developed N-R code in MATLAB.

Despite of unconstrained network size to be solved, several limitations of PSAT were found during the testing stage. 1] Inefficient PV-PQ bus type switching logic is applied. Probably, reverse switching logic (5) is not used and the need for convergence is requested to activate forward switching logic (4). As a result, unnecessarily more PV buses are being switched permanently to PQ. Furthermore, the switching logic completely fails to switch PV buses to PQ for larger systems with high numbers of PV buses. 2] Nominal voltages must be defined in the input data file or the error message 'Divergence - Singular Jacobian' is obtained during the simulation. This seems to be entirely illogical since nominal voltages should not be necessary for the 'in per units defined' problem. 3] It seems that no advanced stability techniques are applied for the N-R method in PSAT because of severe numerical oscillations appearing in several studies. 4] PSAT intentionally neglects transformer susceptances and thus causes errors in final load flow results. A column for shunt susceptances is available for power lines only. For transformers, this column is filled with zeros by default.

Under these limitations, load flow results show very good congruity between the author-developed N-R method and PSAT. Higher total numbers of iterations are needed by PSAT due to missing stability technique(s). Also, CPU times are higher in PSAT due to combining the codes with other analyses and related tool features.

As an example, the load flow and voltage stability analysis of the IEEE 14-bus system is done by PSAT (Figs. 5-9).

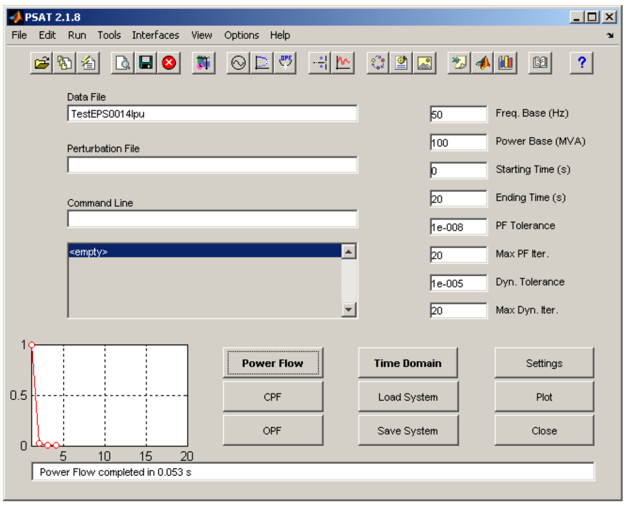

Fig. 5. GUI in PSAT for the load flow analysis of IEEE 14-bus system.

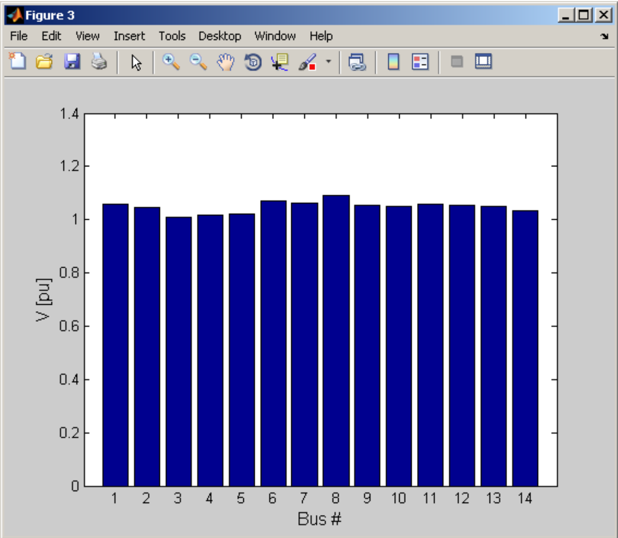

Fig. 6. Final voltage magnitudes of IEEE 14-bus power system in PSAT.

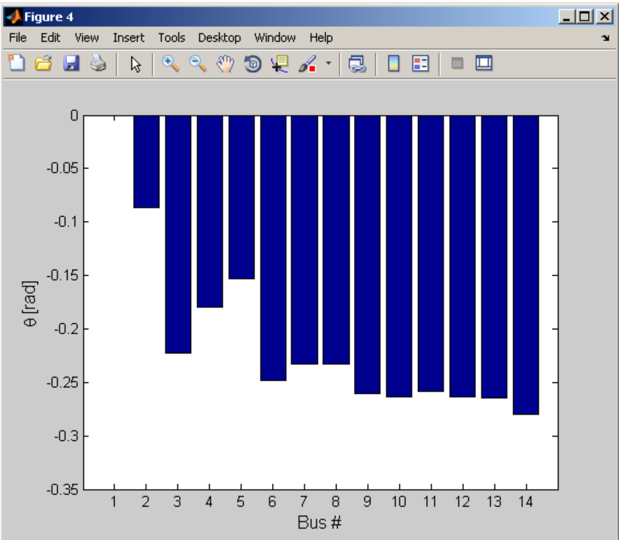

Fig. 7. Final voltage angles of IEEE 14-bus power system in PSAT.

For voltage stability studies, PSAT contains the advanced CLF algorithm, which is combined with contingency and OPF analyses. Load flow data are extended by two matrices specifying the sets of PQ and PV buses, where the loads and generations are to be increased (different increase rates are possible). Thus, various loading scenarios of the system can be modelled.

The CLF code is then started via a specialized window (Fig. 8). Calculation can be adjusted by the user for better computational performance - e.g. by setting a more suitable step size, maximum number of V-P points or by checking the option for controlling voltage, flow or var limits. PSAT offers two CLF methods - perpendicular intersection (PI, as in Fig. 1) and local parametrization (LP). Three stopping criteria are available: The complete Nose Curve (computing both stable/unstable parts of the V-P curve), Stop at Bifurcation (when singular point exceeded) and Stop at Limit (when voltage/flow/point limit hit).

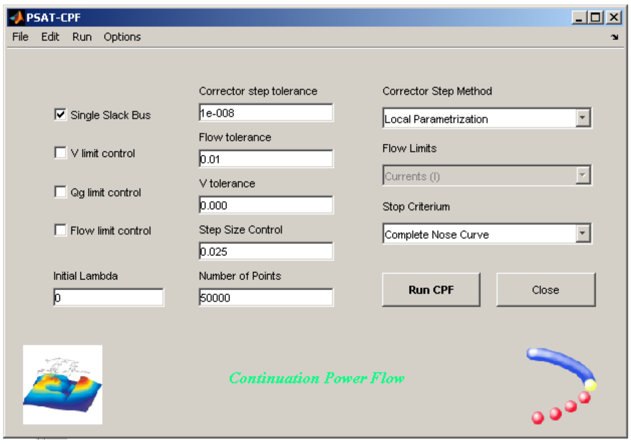

Fig. 8. Settings for the CLF analysis when solving the IEEE 14-bus power system.

The CLF algorithm in PSAT is defined so, that power increases are realized by adding a power increment (loadability factor multiplied by the increase rate) to the base-case loading, i.e. initial λ is zero. In the author-developed Cycled N-R and CLF codes, power increases are performed by multiplying the base-case loading with λ. Therefore, the maximum loadability in PSAT must be increased by unity when comparing both codes.

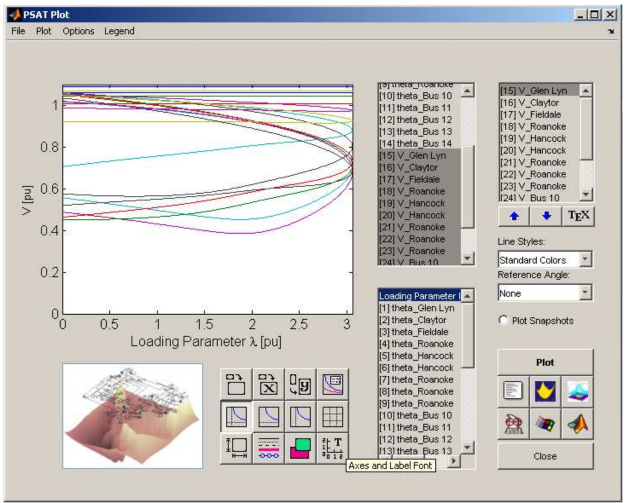

Fig. 9. Nose curves for all network buses of the IEEE 14-bus test system in PSAT.

In Tab. II., voltage stability results for medium-sized IEEE test systems are provided by the author-developed Cycled N-R and compromise CLF codes. These outputs are compared to those obtained by PSAT - see Tab. III. In PSAT, both of the CLF modes were tested (i.e. PI with step 0.025 and LP with default step 0.5).

TABLE II.
VOLTAGE STABILITY ANALYSIS OF MEDIUM-SIZED IEEE TEST SYSTEMS (CYCLED N-R VS. COMPROMISE CLF)

L+G	**Cycled N-R code**			**Compromise CLF code**		
Case	λ_{max} [-]	pts	CPU [s]	λ_{max} [-]	pts	CPU [s]
IEEE9	2.485393	74	0.5460	2.485382	84	0.2964
IEEE13	4.400579	148	0.6708	4.400577	112	0.3120
IEEE14	4.060253	137	0.8268	4.060252	92	0.3276
IEEE24	2.279398	61	0.6396	2.279398	58	0.2496

IEEE30	2.958815	88	0.8112	2.958814	57	0.2964
IEEE35	2.888962	91	0.6864	2.888950	107	0.3432
IEEE39	1.999203	57	0.7644	1.999202	30	0.2184
IEEE57	1.892091	47	0.8892	1.892089	92	0.4836
IEEE118	3.187128	100	1.6536	3.187128	66	0.5772

TABLE III.
VOLTAGE STABILITY ANALYSIS OF MEDIUM-SIZED IEEE TEST SYSTEMS (PSAT - PI VS LP MODE)

L+G	**PSAT - PI mode**			**PSAT - LP mode**		
Case	λ_{max} [-]	pts	CPU [s]	λ_{max} [-]	pts	CPU [s]
IEEE9	2.481220	7	0.2093	2.482000	13	0.3241
IEEE13	4.390420	13	0.3292	4.399570	20	0.4832
IEEE14	4.060100	18	0.4098	4.059420	19	0.4939
IEEE24	2.277550	10	0.2600	2.278670	16	0.4313
IEEE30	2.958550	16	0.8761	2.958250	20	1.5023
IEEE35	2.872940	16	1.1242	2.878420	10	0.2940
IEEE39	1.999110	11	0.2932	1.997840	12	0.3692
IEEE57	1.891920	12	0.9089	1.892090	26	3.9090
IEEE118	3.187100	613	19.1693	3.187120	82	19.7464

Theoretical values of λ_{max}, numbers of stable V-P points and CPU times in seconds are provided for comparison. For all voltage stability studies in PSAT, identical power increase rates were considered. Only the L+G scenario was examined, logics for var limits were deactivated.

Both of PSAT modes showed only average accuracy with satisfiable numbers of V-P points and rather lower computational speed. The LP mode was computationally more time-consuming, but needed lower numbers of V-P points and usually provided more accurate results. The compromise CLF code provided the best combination of solution accuracy and CPU time requirements in each of the cases. Although higher numbers of V-P points were needed, CPU times were still significantly smaller than those in PSAT due to optimized sparse programming applied. Identical conclusions can be made when mutually comparing CLF and Cycled N-R codes.

VIII. CONCLUSION

For solving voltage stability problems, both the Cycled N-R and CLF codes were programmed and comprehensively tested on a broad range of test power systems in MATLAB environment. Various stability techniques, step size approaches and numerical settings were applied and used to upgrade their performance in order to find the algorithm with fair compromise between calculation speed and solution accuracy. The results were compared with outputs obtained from PSAT. The studies imply that the best technique (i.e. best combination of precision level and CPU requirements) is the CLF algorithm with compromise step size settings programmed by Author in MATLAB. However, final technique can be applicable in practice only for off-line planning and development studies of electric power systems. For real-time evaluations of system's voltage stability, a more robust algorithm with minimized numbers of stable V-P points must be developed. Therefore, follow-up research activities will be concentrated especially on this area of interest.

ACKNOWLEDGMENT

This paper has been supported by the European Regional Development Fund and the Ministry of Education, Youth and Sports of the Czech Republic under the Regional Innovation Centre for Electrical Engineering (RICE), project No. CZ.1.05/2.1.00/03.0094. This work has been also sponsored by Technology Agency of the Czech Republic (TACR), project No. TA01020865 and by student science project SGS-2012-047.

REFERENCES

[1] P. Kundur, *Power System Stability and Control*, McGraw-Hill, 1994.

[2] C. Canizares, A.J. Conejo and A.G. Exposito, *Electric Energy Systems: Analysis and Operation*, CRC Press, 2008.

[3] V. Ajjarapu, *Computational Techniques for Voltage Stability Assessment and Control*, Springer, 2006.

[4] I. Dobson, T.V. Cutsem, C. Vournas, C.L. DeMarco, M. Venkatasubramanian, T. Overbye and C.A. Canizares, "Voltage Stability Assessment: Concepts, Practices and Tools - Chapter 2," *Power System Stability Subcomittee Special Publication, IEEE Power Engineering Society*, 2002.

[5] J.H. Chow, F.F. Wu and J.A. Momoh, *Applied Mathematics for Restructured Electric Power Systems - Optimization, Control and Computational Intelligence*, Springer, 2005.

[6] I. Šmon, M. Pantoš and F. Gubina, "An improved voltage-collapse protection algorithm based on local phasors," *Electric Power Systems Research 78 (2008)*, pp. 434-440, 2008.

[7] Y. Gong and N. Schulz, "Synchrophasor-Based Real-Time Voltage Stability Index," *Proceedings of PSCE conference*, pp. 1029-1036, 2006.

[8] J.J. Grainger and W.D. Stevenson, *Power System Analysis*, McGraw-Hill, 1994.

[9] M. Crow, *Computational Methods for Electric Power Systems*, CRC Press, 2002.

[10] J. Veleba, "Application of Continuation Load Flow Analysis for Voltage Collapse Prevention," *Journal Acta Technica*, vol. 57, pp. 143-163, 2012.

[11] P. Zhu, "Performance Investigation of Voltage Stability Analysis Methods," PhD. thesis, Brunel University of West London, 2008.

[12] Homepage of PSAT. [Online]. Available: http://www3.uclm.es/profesorado/federico.milano/psat.htm. [Accessed: 3 Mar. 2013].

[13] Homepage of MATPOWER. [Online]. Available: www.pserc.cornell.edu/ matpower/. [Accessed: 23 Dec. 2012].

[14] Z. Tate, "Initialization Schemes for Newton-Raphson Power Flow Solvers," [Online]. Available: http://grb.physics.unlv.edu/~zbb/files/upload/29UV3GPCVQWO6ERVE9RABFQ5M.pdf. [Accessed: 15 Apr. 2010].

[15] J. Veleba, "Acceleration and Stability Techniques for Conventional Numerical Methods in Load Flow Analysis," *Proceedings of ELEN conference*, pp. 1-10, 2010.

THE AUTHORS

Jan Veleba received his Master degree in Power Engineering at the University of West Bohemia, Pilsen, Czech Republic in 2008. Currently, he is the PhD. student at the Department of Electrical Power Engineering and Environmental Engineering at the University of West Bohemia in Pilsen, Faculty of Electrical Engineering. His main research activities concern load flow, voltage-power control and voltage stability analyses of particularly larger and more complex electric power systems.